

# Description of the Deficit Irrigation Economics Model II

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## General Comment

This spreadsheet is a model that is described in detail in: Trout, T.J. and D.T. Manning (2019) An economic and biophysical model of deficit irrigation. *Agron. J.* 111:1–12. doi:10.2134/agronj2019.03.0209 , and Trout et al. (2020) Deficit Irrigation Strategies for the Western U.S.. *Trans ASABE* (in review). Those publications define terms and develop the equations used in the spreadsheet.

## General Instructions

"This spreadsheet presents a biophysical and economic model of deficit irrigation. This spreadsheet differs from USDA Ag Data

Commons site: DOI: 10.15482/USDA.ADC/1504421 in that it includes water limiting conditions and does not include income from water leasing. Parameters are input into the yellow highlighted cells of each worksheet. Parameters are defined in Notes attached to cells. Parameters input or calculated in previous worksheets are copied into and displayed in each worksheet. Relationships are graphed. Final output is the net income (NI) for a given set of biophysical and economic parameters. ""Optimum"" solutions are shown."

## WPF Model - Water Production Function and Water Productivity (3 input parameters)

The normalized Water Production Function (WPF) model is based on normalizing both the yield and ETc relative to the maximum Yield ( $Y_m$ ) and maximum (potential) ETc ( $ET_m$ ). The normalized WPF is modelled as a 2nd degree polynomial:  $Y_R = A + Bx + Cx^2$  where  $x$  = the relative ET. The coefficients of the normalized WPF are derived based two points on the WPF and the degree of curvilinearity, represented by the coefficient of the squared term ( $C$ ). The upper point is 1,1 (full ET and maximum yield). A common scenario for the second point is  $Y_R = 0$  at  $x = x_0$  (the lower point on the x intercept). The  $C$  coefficient can vary from 0, which produces a linear WPF, to negative values that produce concave downward WPFs. The right-side graph shows the effect of  $C$  on the WPF shape. With these assumptions, the remaining polynomial coefficients ( $A$  and  $B$ ) are derived, and the WPF is plotted (graph (a)).

Through algebraic manipulation of the normalized WPF, the water productivity ( $WP = Y/ET$ ) and the marginal WPF (derivative of WPF) are derived and shown on graphs (b) and (c). The relative ET value is determined that gives the maximum WP, and the minimum relative ET that produces  $WP > 1$  (the lower end of the range for which  $WP > 1$ ). These points are shown on the WPF graph. For deficit irrigation to be potentially beneficial, the marginal WPF should be less than or equal to 1 at  $x = 1$  ( $C > (1-B)/2$ ).

## IrrReq - Irrigation Requirement (12 Input Parameters)

The Irrigation Requirement is based on ET being met by the sum of three sources of water: Irrigation Supply (I), Effective Seasonal Precipitation (Re), and seasonal change in Soil Water Storage (Se) (final minus initial). Each source has an associated "effectiveness" or use efficiency, or that portion of the water source that is used for ETc. The three efficiencies,  $E_i$ ,  $E_r$ , and  $E_s$ , vary with the relative ET and thus can be derived as a function of  $x$ . All three efficiencies are assumed to be 1.0 (100%) at low ET (large deficit), and decrease to a minimum value at  $ET_m$  ( $x=1.0$ ). The relative decrease is assumed to be linear or concave and is modeled as a second-degree polynomial. The amount of effective rainfall and water from storage is then calculated along with the deficit, irrigation supply required to meet the target ET, and total water supply. The deficit is calculated as ET minus effective precipitation and storage. The required irrigation supply is the deficit divided by the irrigation efficiency. The total water supply is calculated as the sum of R, S, and I.

Precipitation Efficiency,  $E_r$ : User inputs an assumed precipitation efficiency at  $ET_m$ ,  $E_{r1}$ ; the relative ET value,  $x_r$ , below which  $E_r = 1$ ; and a quadratic coefficient,  $G$ , that defines the curvilinearity of the relationship between these two points. The other two polynomial coefficients are calculated. The quadratic coefficient,  $G$ , will be  $\leq 0$ , with 0 providing a linear relationship.  $E_r$  values must be limited to  $\leq 1.0$ .

Storage Efficiency,  $E_s$ : User inputs the portion of the potential off-season soil water storage that is used by the crop at  $ET_m$ ,  $E_{s1}$ ; the relative ET value,  $x_s$ , below which all of the off-season effective precipitation is used ( $E_s = 1$ ); and a quadratic coefficient,  $K$ , that defines the curvilinearity of the relationship between these two points. The other two polynomial coefficients are calculated. The quadratic coefficient,  $K$ , will be  $\leq 0$ , with 0 providing a linear relationship.  $E_s$  values must be limited to  $\leq 1.0$ .

Irrigation Efficiency,  $E_i$ : User inputs an assumed irrigation efficiency at  $ET_m$ ,  $E_{i1}$ ; the relative ET,  $x_i$ , below which  $E_i = 1$ ; and a quadratic coefficient,  $N$ , that defines the curvilinearity of the relationship. The other two polynomial coefficients are calculated. The quadratic coefficient,  $N$ , will be  $\leq 0$ , with 0 providing a linear relationship.  $E_i$  values must be limited to  $\leq 1.0$ .

Deficit (Irrigation Requirement), IR, and Irrigation Supply, I: These are the remaining deficit (ET minus effective precipitation and storage), and the Irrigation amount required to achieve the target ET. User inputs the anticipated in-season (planting to harvest) precipitation amount, R; the anticipated effective (infiltrated and not evaporated) off-season precipitation, S; and a seasonal maximum ET,  $ET_m$ . The irrigation requirement is calculated as the ET ( $ET_m * x$ ) minus the effective seasonal precipitation, Re ( $R * E_r(x)$ ) and minus the effective storage, Se ( $S * E_s(x)$ ), all of which are functions of the relative ET,  $x$ . The required irrigation supply is then  $IR/E_i$ . These are plotted vs.  $x$ .

Production Functions: The relative yield,  $Y_R$ , is plotted as a function of crop ETc (WPF), IR, I (IWPF) and total water supply (TWPF). This demonstrates the effect of precipitation, soil water storage, and irrigation efficiency on the crop production function. Note that the IR and I production function intercepts are positive if  $Re + Se$  is larger than  $ET_m * x_0$ ; and their slopes are less than that of the ET WPF.

A quadratic equation is fit to the IWPF relationship (in terms of relative yield,  $Y_R$ ). Three points used to calculate the coefficients ( $k_0$ ,  $k_1$ ,  $k_2$ ) are yield with no irrigation (0,  $k_0$ ),  $I_s$  at  $ET_m$  ( $I_m$ , 1) and an intermediate point on the curve ( $z * I_m$ ,  $Y_{Rz}$ ). Since the equations are written in terms of relative ET and relative  $Y$ , the relative ET for no irrigation (rainfed production) must first be determined iteratively (cell

A71). The value is correct when  $I_s = 0$ .  $k_0$  is the predicted rainfed yield at this rainfed relative ET. Then an intermediate point is selected on the IWPF curve between rainfed ET and  $ET_m$  (cell A72). In cell A72, the intermediate point is 2/3 the way between rainfed ET and  $ET_m$ . The relative Y is calculated at this intermediate point.  $IM$  is the full irrigation application that meets  $ET_m$  (cell B77). The intermediate point is then converted from relative ET to relative IS by dividing by  $IM$ . Then  $k_2$  and  $k_1$  are calculated from these three points (see App 1 of Trout et al. (2020)). The graph shows the model fit (IWPFm). The  $Y_R(I)$  are converted to actual yield by multiplying the relative yield by  $Y_m$ . A Power Curve (Cobb-Douglas) model is also developed (Martin et al., 1989).

### **UnlWaterNI: Net Income Economic Model based on Adequate Irrigation Supply (unit area basis) (5 input parameters)**

This worksheet calculates the net income, NI, for varying irrigation levels when the water supply is adequate for full irrigation. The worksheet uses the models for  $Y_R(x)$  and  $I(x)$  developed in the previous two worksheets (parameters are copied into cells R5 - U19). Crop price and production costs are input into highlighted cells in Row 6. NI is calculated for incremented relative ET values in Rows 25 - 125 based on Eq. 1 (Trout et al., 2020). Revenue, Costs, and NI are plotted in the graph. All NI calculations are per unit area (ha). The  $I$  that maximized NI,  $I_{op}$ , is estimated numerically and from Eq. 4.

$Y_m$ , Yield price and three costs of production are input into the highlighted cells in Row 6. The table calculates each component of NI at increasing relative ET amounts,  $x$ . The revenue and cost functions are plotted, along with the NI function. The level of  $I$  that provides the maximum NI is estimated numerically and presented in cells K31:L33. The  $I_{op}$  is also estimated from the quadratic IWPF model (cell D13). Note that the water price is given in  $m^3/ha$  so the irrigation amount (ha-mm/ha) must be multiplied by 10 to calculate the water cost.

### **WatLimNI: Net Income Economic Model based on limited water supply (unit area basis) (10 input parameters)**

This worksheet calculates the net income, NI when the water supply is limited. Income and costs from the unirrigated area are included. The worksheet uses the models for  $Y_R(x)$  and  $I(x)$  developed in the WPF and IrrReq worksheets (parameters copied into Cells R5 - U19). Crop prices and production costs are input into highlighted cells in Row 6. NI calculations are made in Rows 25 - 125 for incremental values of relative ET,  $x$ . NI trends are plotted in the graphs vs.  $x$ ,  $A_i$ , and  $I$ . NI is calculated per unit land area and calculations are based on relative area that is irrigated,  $A_i$  and unirrigated,  $A_r$ .

Yield at full irrigation,  $Y_m$ , and the available irrigation water supply,  $W$ , are input into highlighted cells in Row 6. Yield price and three costs of production for the irrigated crop are input into the highlighted cells in Row 6 and applied to  $Y$  and  $I$  to calculate NI per unit area for the irrigated crop. Likewise, a price and two production costs are input into the highlighted cells in Row 6 and applied to the expected rainfed yield,  $Y_r$ , to calculate NI per unit area of the un-irrigated crop. The expected yield of the unirrigated crop is input into cell I19. The worksheet provides an expected rainfed crop yield (cell N14)

based on the y-intercept ( $I = 0$ ) of the IWPF of the irrigated crop. The worksheet then calculates the expected (constant) NI per unit area of the rainfed crop (cells I20, I21).

Based on the available water supply and the irrigation depth, the worksheet calculates the portion of the relative area that is irrigated,  $A_i$ , and the remainder that is unirrigated,  $A_r$ , for each relative ET,  $x$ . These fractions are multiplied by the NI per unit area of the irrigated and unirrigated land, respectively, and summed to determine the total NI per unit area. This value can be multiplied by the total area to determine the total NI.

The revenue and cost functions are plotted, along with the NI function vs.  $x$ ,  $A_i$ , and  $I$ . The level of  $I$  that provides the maximum NI is estimated numerically and presented in cells U92-V96. The  $I$  value that provides the maximum NI is also estimated by the quadratic model of the  $Y(I)$  equation (cell D24) and the NI for this  $I$  value is calculated (Row 24). Note that the water price is given in  $\text{m}^3/\text{ha}$  so the irrigation amount ( $\text{ha-mm}/\text{ha}$ ) must be multiplied by 10 to calculate the water cost. These calculations are in terms of NI per unit area. They can be multiplied by the total area to calculate total NI. These NI calculations are only relevant between  $W < I < I_M$  or equivalently,  $W/I_M < A_i < 1.0$ .